

1. The region bounded by the curve $y = \sin x$, the lines $x = 0$, $x = \pi/2$, $y = 0$ is revolved about the line $x = \pi$. Find the volume of the resulting solid.

4 points

2. Identify the graph of the equation $25x^2 + 4y^2 + 50x - 16y - 59 = 0$. Find its centre, foci, vertices and sketch the graph.

4 points

3. Show that

$$(a) \quad \tanh\left(\frac{x}{2}\right) = \frac{\sinh x}{1 + \cosh x} \quad (x \in \mathbb{R})$$

$$(b) \quad \arcsin x + \arccos x = \pi/2 \quad (0 < x < 1).$$

2+3 points

4. Evaluate the following integrals:

$$(a) \quad \int \frac{5x^2 + x - 2}{(1 + x^2)(3x - 1)} dx$$

$$(b) \quad \int \sqrt{\frac{1 - \cos x}{1 + \cos x}} dx$$

$$(c) \quad \int \tan^2 x \sin^2 x dx$$

$$(d) \quad \int (\arcsin x)^2 dx.$$

5 points each

5. Find the following limits if they exist:

$$(a) \quad \lim_{x \rightarrow \infty} x \ln\left(\frac{x+1}{x-1}\right)$$

$$(b) \quad \lim_{x \rightarrow \infty} \left(\frac{\int_0^x \arctan t dt}{x} \right).$$

(c) Determine whether the improper integral

$$\int_0^{\infty} \frac{dx}{\sqrt{1 + \sinh^2 x}}$$

is convergent or divergent, and if convergent find its value.

3+4+4 points

6. Find the area of the region which is outside the curve $r = 2 + 2 \cos \theta$ and inside the curve $r = 6 \cos \theta$.